5.4b CHI^2 Test: Hypothesis Testing

X^2 Test for Independence

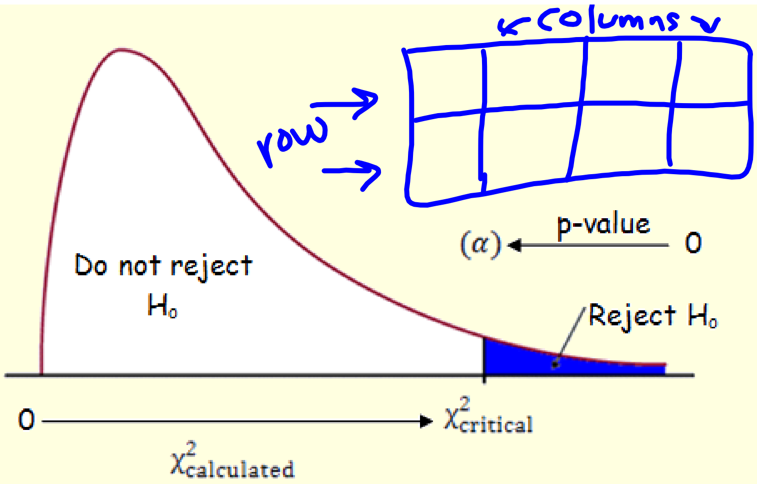
Last class we used the GDC to calculate the χ^2 value for a set of data.

What does the χ^2 test actually measure?

tests how likely it is that an observed distribution is due to chance - also called "goodness of fit test" - test for independence

Small χ^2 values mean that there is not much difference between observed and expected values. This could mean the variables are independent.

Large χ^2 values mean that there is variance between observed and expected values. This could mean the variables are not independent.



Once you have calculated the χ^2 statistic, you have to compare it to a *critical value*. The critical value will always be given to you, but it is determined by:

1. The *degrees of freedom* of the data.
   More degrees of freedom lead to higher $\chi^2_{critical}$ values.

degrees of freedom = (rows - 1)(columns - 1)

$(2-1)(4-1) = 3$

2. The *significance level* (α) desired from the test.

The significance level (α) indicates how willing we are to be wrong about the independence of the two variables. It corresponds to how sure we want to be that our findings are significant.

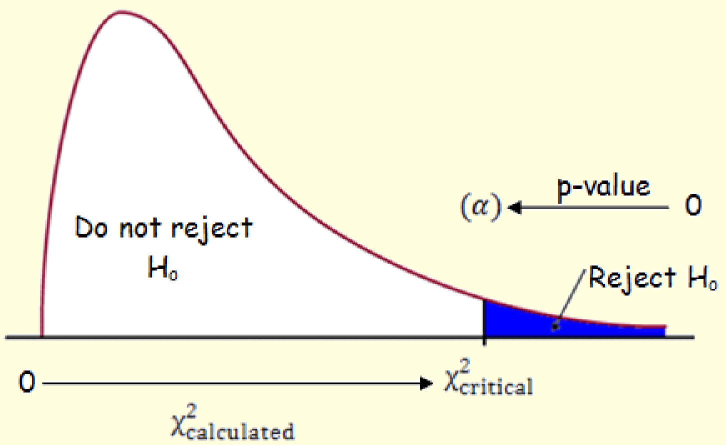
A significance level $\alpha = 0.10$ means that we are 90% confident about significance.

A significance level $\alpha = 0.05$ means that we are 95% confident about significance.

A significance level $\alpha = 0.01$ means that we are 99% confident about significance.

Higher confidence levels lead to higher χ^2_{critical} values.

Therefore, lower significance levels also lead to higher χ^2 critical values



If $\chi^2_{calculated}$ __<__ $\chi^2_{critical}$ value, then we __do not reject__ the null hypotheses H_0.

If $\chi^2_{calculated}$ __>__ $\chi^2_{critical}$ value, then we __reject__ the null hypotheses H_0.

Example

Three hundred students were observed wearing colored T-shirts.

	Black	White	Red	Blue	Totals
Male	48	12	33	57	150
Female	35	46	42	27	150
Totals	83	58	75	84	300

Calculate the degrees of freedom:

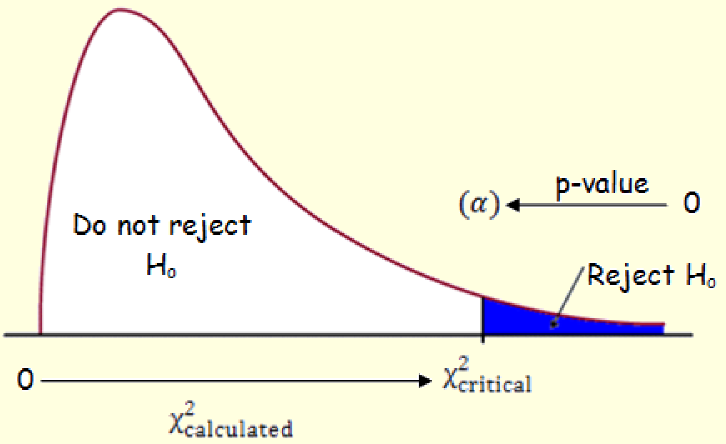
$(2-1)(4-1) = 3$

At the 5% level of significance, the χ^2_{critical} value is 7.815.
What can you conclude about gender and T-shirt color? Justify your answer.

$\boxed{\chi^2 = 33.76} > 7.815$

reject H_0 → greater than critical

We have enough evidence to reject the null hypothesis and accept H_1 (alt. hypothesis) T-shirt color and gender are not independent.



Another way to determine the validity of a hypothesis is to consider the *p-value*. The p-value is the probability that we have incorrectly rejected the null hypothesis. P-values are measured from the right, and compared to the level of significance, (α).

If the p-value $<$ significance level, then we reject H_0.

If the p-value $>$ significance level, then we do not reject H_0.

Example

One hundred people were interviewed outside a chocolate shop to find out which flavor of chocolate crèmes they preferred. The observed results are given in the table, classified by gender.

	Strawberry	Coffee	Orange	Vanilla	Totals
Male	23	18	8	8	57
Female	15	6	12	10	43
Totals	38	24	20	18	100

Calculate the degrees of freedom:

$(2-1)(4-1) = 3$

Write down the p-value:

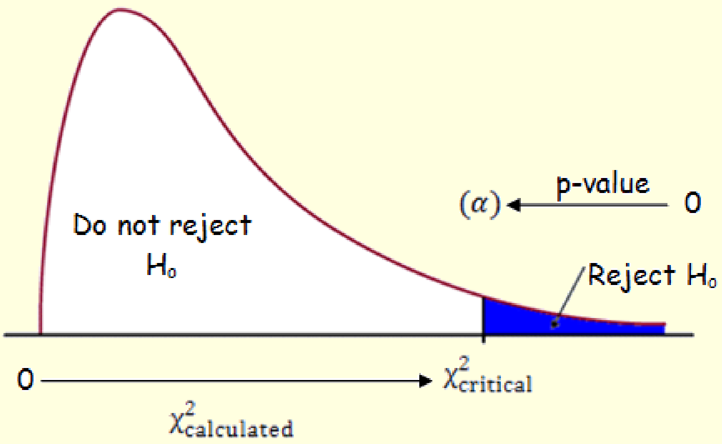
$p = 0.0758$

At the 5% level of significance, what can you conclude about gender and chocolate preferred? Justify your answer.

$0.05 < 0.0758$

We fail to reject the H_0 (null hypothesis).

We have enough evidence to conclude that gender and preferred chocolate are independent.


Do not reject
H_0
(α)
p-value
0
Reject H_0
0
$\chi^2_{critical}$
$\chi^2_{calculated}$

Homework: 238-240:1-6 all
(even, use chi-squared; odd, use p-value)